A Case Study of Two Groups of Elementary Prospective Teachers' Experiences in Distinct Mathematics Content Courses

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This multiple case study examined two groups of elementary prospective teachers (*n*=12) completing distinct mathematics content courses. Data were collected via two belief surveys, one content knowledge assessment, and individual interviews. The findings revealed differences in specialized content knowledge and mathematical beliefs between the two groups upon completion of the teacher preparation program. Also, they characterized their experiences with knowing, learning, and teaching mathematics during the courses in dissimilar ways. The findings offer insights into course content, pedagogical approaches, and learning activities that prompted motivation, learning, and change. Notably, elementary prospective teachers understanding the applicability of the mathematics they are learning to their chosen career path is paramount.

Improving the mathematical knowledge of elementary teachers is of significant concern in the U.S., especially in light of the increased rigor and depth of the mathematics for students in the *Common Core State Standards for Mathematics* (CCSS-M, NGACBP & CCSSO, 2010). Accordingly, many institutions of higher education now require specialized mathematics content courses for elementary prospective teachers. These courses signify a key advancement in teacher preparation, grounded in the realization that teachers should study the mathematics they teach in depth, and from the viewpoint of the teacher (Conference Board of the Mathematical Sciences [CBMS], 2012). Notably, in *The Mathematical Education of Teachers II*, the CBMS (2012) stresses, "Programs designed to prepare elementary teachers should include 12 semester hours focused on careful study of mathematics associated with the CCSS (K-5 and related aspects of 6-8 domains) from a teacher's perspective" (pp. 31). However, such courses hold challenges inherent to elementary teachers, including the tendencies of this population to have negative affect toward mathematics, such as a dislike and avoidance of the subject (Bekdemir, 2010; Philipp, 2007), and to espouse traditional, procedural views on what it means to know and do mathematics and how it is taught (CBMS, 2012). These difficulties are compounded by the perspective of some who think they do not need to learn more mathematics, as their experiences with mathematics thus far have provided them the content knowledge needed for teaching in the elementary classroom. Given these constraints, careful consideration of learning experiences and outcomes in specialized mathematics content courses is key to understanding efficacious ways of both developing content knowledge and changing affect.

Hence, this study took place in an elementary teacher preparation program that had changed to align with university system mandates by including four 3-hour courses in mathematics content for elementary teachers. Effects of the required courses on student progression in the program became of immediate concern. Over one 4-semester period, 25% of students did not complete or pass one or more of these mathematics courses. In response to this troubling trend, a group of students enrolled as a cohort in a one-time sequence of the four content courses having specific foci on the perspectives found in the National Council of Teachers of Mathematics (NCTM, 2000) *Principles and Standards for School Mathematics* and the CCSS-M (e.g., problem-based pedagogy; dialogic classroom discourse; explanation and justification, representations, applications, and connections in mathematics) and the development of *specialized content knowledge* (SCK) for teaching in the elementary classroom. This experimental sequence is referred to here as the "alternate courses"; the other group of students in this study participated in what is referred to here as the "typical courses". This inquiry explored these two groups of students' mathematical beliefs and content knowledge, as well as their perspectives on knowing, learning, and teaching mathematics as experienced in the context of the courses.

Theoretical Perspectives and Related Research

Teachers require deep and broad knowledge of mathematics to be effective in their teaching (Hill, 2010), particularly as they "are in the unique position of having to professionally scrutinize, interpret, correct, and extend [student] knowledge" (Ball, Hill, & Bass, 2005, p. 17). In recent times, there have been multiple efforts to define the exact *mathematical knowledge for teaching* (MKT) (Ball & Forzani, 2010; Ball, Thames, & Phelps, 2008; Hill, 2010), and researchers have proposed a SCK characterized as "mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (Ball et al., 2008, p. 399). Examples of SCK include representing numbers and operations with pictures or manipulatives, examining and generalizing from non-standard solution methods, and providing explanations for mathematical ideas or procedures (Schilling & Hill, 2007).

In addition to content knowledge as an important teacher competency, across many years studies have revealed a well-established link between teachers' mathematical beliefs and their instructional practices (Philipp, 2007; Raymond, 1997). Beliefs are considered to be the cognitive set of psychological understandings, premises, or propositions through which interpretations are made of the surrounding world (Philipp, 2007). The influence of affect, which is comprised of beliefs, attitudes, and emotions, on teacher learning during university mathematics content courses is considerable. The sizeable amount of research on teacher affect testifies to elementary prospective teachers' tendencies to enter their preparation programs with affective states that are less than optimal for both their own learning and their future as teachers of mathematics (Philipp, 2007). For example, many exhibit high levels of mathematics anxiety (Bekdemir, 2010). If these affective states are not addressed (or worse ignored) during mathematics content courses, there is little hope for change.

Mathematics content courses offer an important venue, and typically one of the last before assuming teaching positions in schools, for elementary prospective teachers to learn mathematics in ways that alleviate negative affect toward mathematics and ideally prompt shifts in a positive direction. Two belief constructs relevant to this study include: pedagogical beliefs (i.e., one's beliefs about teaching and learning) and teaching efficacy beliefs (i.e., beliefs in one's abilities to teach mathematics effectively and influence student learning).

In order to develop the mathematical knowledge for teaching in the elementary classroom and support positive shifts in mathematical beliefs, several features of effective teacher preparation programs have been identified (Authors 2007, 2009, 2012; CBMS, 2012; Philipp, 2007; Sowder, 2007). In particular, mathematics courses should examine in depth, and from a teacher's view, the vast majority of K-5 mathematics, and its connections to PreK and middle school mathematics. Further, coursework should provide time and opportunities to think about, discuss, and explain mathematical ideas, while developing mathematical habits of mind and furthering mathematics as a sense-making enterprise. In addition, program design should include a seamless blend of study of the mathematical content and teaching methods, and departments of education and mathematics should collaborate, including mathematics educators and mathematicians, in the preparation of elementary prospective teachers. Specific methods for prompting teacher learning in mathematics include studying children's thinking, using reform-oriented curricula and cognitively demanding instructional tasks, emphasizing problem solving and other mathematical processes, examining case studies of teaching and learning, and relating coursework to K-12 classrooms (Lannin & Chval, 2013; Philipp, 2008; Philipp et al., 2007; Sowder, 2007).

Research Questions

This case study examined two groups of students (i.e., elementary prospective teachers) in distinct mathematics content courses and was guided by the following research questions:

1. What are the mathematical beliefs and content knowledge of two groups of students in distinct mathematics content courses?
2. What are their perspectives on knowing, teaching, and learning mathematics as experienced in these courses?

Methodology

The design of this study incudes a descriptive, holistic multiple-case approach (Yin, 2003). The cases were the two groups of students in distinct mathematics content courses, and the purpose of the study was to provide a *thick description* of each.

Participants and Setting

The participants consisted of 12 randomly selected students (11 females, 1 male) enrolled in an Early Childhood Education (grades PreK-5) teacher preparation program at a large, urban university in the southeastern U.S. The program was two years in duration and included three semesters of education courses with concurrent two-day-per-week field placements, followed by a full semester of student teaching. All participants had completed four mathematics content courses for elementary teachers, including Foundation of Number and Operations, Geometry and Spatial Reasoning, Introduction to Probability and Statistics, and Algebraic Concepts, as well as one mathematics teaching methods course. At the time of registration for the first mathematics content course, the participants chose a course section based on convenience and were not aware that a section would offer an alternate experience. As an admittance requirement, all students completed the Foundation of Number and Operations course prior to entering the teacher preparation program. The remaining three mathematics content courses were completed at various times prior to the student teaching semester. The mathematics methods course, taught in the Early Childhood Education Department, was completed during the second semester of the program. The composition of prospective teachers in the mathematics content courses was not the same as those in the mathematics methods courses.

Six students experienced the "alternate courses" taught by an instructor in the Early Childhood Education Department, and six of the students experienced the "typical courses" taught by instructors in the Mathematics Department. The nature of the course experiences is best interpreted from the responses of the study participants, but is supported by a syllabi analysis which revealed differences in topics, textbooks, and assignments. For example, the textbooks for the alternate courses included sections on children's thinking related to mathematical ideas and cases of teaching and learning in the elementary classroom; sections such as these were not evident in the textbooks used in the typical courses. Assignments in the typical courses were largely comprised of tests, homework, and quizzes. The alternate courses included similar assignments plus assessments/analyses of children's thinking and development/analyses of worthwhile mathematical tasks. Lastly, the syllabi analysis showed that overall the typical courses contained a longer list of topics, while the alternate courses focused on fewer topics. This syllabi analysis provides insights into the intended learning goals and curriculum, as well as a general context for the comments of the students.

Data Collection

Quantitative data were collected via two belief surveys and one knowledge assessment, and qualitative data were collected via individual interviews. The instruments were administered on campus, and the semi-structured, individual interviews took place either in a researcher's office on campus or the student teaching school. Data collection occurred predominantly during the student teaching semester.

The Mathematics Beliefs Instrument (MBI) is a 48-item Likert scale instrument designed to assess teachers' beliefs about the teaching and learning of mathematics and the degree to which these beliefs are cognitively aligned (Peterson, Fennema, Carpenter, & Loef, 1989, as modified by the Cognitively Guided Instruction Project). The three subscales include: (b) role of the learner (Learner), (b) relationship between skills and understanding (Curriculum), and (c) role of the teacher (Teacher). The Learner subscale contains 15 items that assess the degree to which teachers believe that children can construct their own mathematical knowledge. The 16-item Curriculum subscale examines the degree to which teachers believe that mathematics skills should be taught in relation to understanding and problem solving. The 17 items on the Teacher subscale address the extent to which teachers believe that mathematics instruction should be organized to facilitate children's construction of knowledge. The instrument uses a Likert scale with five response categories (strongly agree, agree, uncertain, disagree, and strongly disagree), with higher scores indicating beliefs that are more cognitively oriented. These subscales have high reliability (Chronbach's alpha =.89 for Learner, .80 for Curriculum, and .90 for Teacher) and represent independent constructs based on confirmatory factor analysis.

The Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) consists of 21 items, 13 on the Personal Mathematics Teaching Efficacy (PMTE) subscale and 8 on the Mathematics Teaching Outcome Expectancy (MTOE) subscale (Enochs, Smith, & Huinker, 2000), consistent with the two-dimensional aspect of teacher efficacy. The PMTE subscale addresses the prospective teachers' beliefs in their individual capabilities to be effective mathematics teachers. The MTOE subscale addresses the prospective teachers' beliefs that effective teaching of mathematics can bring about student learning regardless of external factors. The instrument also uses a Likert scale with five response categories, with higher scores indicating greater teaching efficacy. These subscales have high reliability (Chronbach's alpha = .88 for PMTE and .81 for MTOE) and represent independent constructs based on confirmatory analysis.

The Learning Mathematics for Teaching (LMT) instrument examines teachers' SCK for teaching mathematics (Hill, Schilling, & Ball, 2004). It assesses this knowledge by posing mathematical tasks that reflect what teachers encounter in the classroom, such as assessing students' work, representing mathematical ideas and operations, and explaining mathematical rules or procedures. Content knowledge subscales in this instrument include: (a) Number and Operations, (b) Patterns, Functions, and Algebra, and (c) Geometry (Hill, 2004). Content validity was established by mapping items for congruence with the NCTM Standards (Siedel & Hill, 2003). Analysis of reliability indicated alpha coefficients of .79 for the Number and Operations subscale, .75 for the Patterns, Functions, and Algebra subscale, and .85 for the Geometry subscale (G. Phelps, personal communication, October 6, 2006). IRT scaled scores for the LMT are in the range from –3 to +3, with an expected value of zero.

The interview protocol includes six multi-part questions, with sample questions such as: (a) What are your overall impressions of the math courses? What was easy and hard? What did you like and dislike? (b) After taking the math courses, do you feel confident that your content knowledge is sufficient to understand PreK-5 math? Why or why not? and (c) After taking the math courses, do you feel prepared to analyze children's math strategies in grades PreK-5? Why or why not?

Data Analysis

This multiple-case design included analysis of the data within each case. The findings aim to be descriptive, exploring the purpose of the study in its particular context. For the quantitative data, the instruments were considered at the case level by subscale and overall scale.

For the qualitative data, audiotapes of the interviews were transcribed and analysis of the data involved applying the *a priori* codes of *knowing*, *learning*, and *teaching* mathematics as experienced by the two groups of students in the mathematics courses. Drawing upon a framework for three components of teachers' conceptions from Authors (2005), this analysis examined the interviews for statements about what constitutes knowing mathematics, how knowledge is produced through learning processes, and what teaching actions are important to initialize and sustain the learning processes to produce the desired knowledge. Researchers periodically met to discuss findings related to these codes, and this recursive process of discussion and analysis of all interview data initiated development of a coding manual used in subsequent analyses. The researchers then engaged in data reduction by recoding data using the coding manual for guidance.

Results

Quantitative Findings

Mean scores, differences in mean scores, and standard deviations on the MBI and MTEBI (subscales and overall scale) are shown in Table 1. When examining the two sets of scores, all subscale and overall scale mean scores have at least half-point differences in the Likert scale value. These findings suggest those in the alternate courses had pedagogical beliefs more cognitively aligned and stronger mathematics teaching efficacy beliefs at the end of the program. The Learner and Teacher subscales evidenced the largest differences (.87 and .75, respectively) in mean scores, thus these students in the alternate courses more so believed that children can construct their own mathematical knowledge and that instruction should be organized to facilitate that construction. Interestingly, the subscale with the next largest difference in mean score (.70) was the MTOE. When considering these two groups of students, those completing the alternate courses seem to have stronger beliefs their teaching of mathematics positively influences student learning.

Table 1
Means and Standard Deviations for Mathematics Pedagogical and Teaching Efficacy Beliefs as Likert Scale Values

	Typical Courses		Alternate Courses		Both Courses
Subscale and Overall Scores	Means	Standard Deviations	Means	Standard Deviations	Differences in Mean Scores
Learner	3.59	.67	4.46	.49	.87
Curriculum	3.48	.37	4.15	.47	.67
Teacher	3.80	.46	4.55	.32	.75
Overall MBI	3.62	.42	4.39	.31	.77
PMTE	4.08	.23	4.68	.34	.60
MTOE	3.61	.28	4.31	.74	.70
Overall MTEBI	3.90	.13	4.54	.44	.64

Note. MBI = Mathematics Beliefs Instrument; PMTE = Personal Mathematics Teaching Efficacy; MTOE = Mathematics Teaching Outcome Expectancy; MTEBI = Mathematics Teaching Efficacy Beliefs Instrument.

Table 2 provides the IRT mean scores, differences in scores, and standard deviations for these two groups of students on the LMT by subscale and overall scale. Mean IRT scores for these students in the alternate courses were substantially higher on the overall scale and on two of the three subscales (Number and Operations and Geometry), perhaps indicating more developed SCK. The mean IRT scores on the Patterns, Functions, and Algebra subscale were similar for both groups of students. Variability in scores, as indicated by standard deviations, was greater within those in the typical courses overall and on two of the three subscales (Number and Operations and Patterns, Functions, and Algebra). Standard deviations for both groups were comparable on the Geometry subscale.

Table 2

Mean IRT Scores and Standard Deviations for Learning Mathematics for Teaching (LMT) Instrument

	Typical Courses		Alternate Courses		Both Courses
Subscale and Overall Scores	Mean IRT Scores	Standard Deviations	Mean IRT Scores	Standard Deviations	Differences in Mean Scores
Number and Operations	-.686	.98	.116	.28	.802
Geometry	-.373	.54	.529	.47	.902
Patterns, Functions and Algebra	-.345	1.03	-.279	.46	.066
Overall LMT	-.468	.51	.122	.36	.590

Qualitative Findings: Typical Courses

Knowing mathematics. The students characterized mathematics in the typical courses as: procedural knowledge, lacking relevance, and difficult. Specifically, mathematics as procedural knowledge included descriptors such as “formulas,” “step-by-step,” “right and wrong,” “abstract,” “information,” and “definitions,” with little attention to processes in mathematics. Mathematics was typified as a *record of knowledge.*

Additionally, the students frequently spoke of the irrelevance of the mathematics, describing the mathematics as “high school” or “college” level, with little connection to the mathematics in the elementary classroom. The students stated, “There weren’t a lot of things that I could take from there [courses] and take into my classroom . . . The content that we were working on, the ways we were doing it, it was more, middle school, high school, or college,” and “It was not taught as this is what you are going to use as an elementary educator to be good at it, to teach it to children . . . There was never a connection between the course we were taking and its purpose in the classroom as an educator.” A student provided this example: “You are sitting there and you’re like crunching z scores and you’re doing these two page problems. And, you’re wondering where does this come into play with . . . adding and subtracting double digit numbers.” This perceived lack of connection frustrated the students, as one stated, “It was the biggest waste of time like I have ever gone through . . . I need to be able to offer something back to my [elementary] students, and I don’t feel I got anything.”

Interestingly, before completing the courses, the students believed they already had the mathematical knowledge needed for teaching in the elementary classroom; the courses did not challenge their paradigm about SCK for teaching elementary mathematics. For example, a student asserted, “I felt like I came in with the math content knowledge when I came to college . . . I have elementary content when we speak of teaching from [grades] K to 5. I think I came into [the university] with that knowledge.” The students also described the difficulty of the mathematics in the courses, particularly as “hard” and “unattainable.” For example, a student stated, “[the course] was very, very hard for not only myself but a lot of people in my cohort.”

Learning mathematics. In considering the learning and teaching of mathematics in the context of the typical courses, it is noteworthy there was little mention of the learning and teaching of mathematics for elementary students. Learning mathematics was characterized as: rote memorization, a process that occurs via experts, and “passing the course.” The students described their learning through rote memorization as “time-consuming, extensive practice,” “note-taking”, “homework,” and “repetition and regurgitation.” For example, a student characterized learning as, “It was more of the math courses that I’ve had my entire life memorize, regurgitate, memorize, regurgitate… I was taught to memorize facts.” Another asserted:

> The amount of homework that was assigned was overwhelming, and it wasn’t really realistic. We would have almost every problem from the book assigned for homework, and we would be expected to do those problems and get them correct. And, that was a big portion of our grade. And, I probably spent days and days, like I would spend eight hours in one day just doing homework.

Further, a student explained, “It was Power Point presentation put up for three hours . . . I can’t learn from just looking at Power Point slides honestly and then trying to do one hundred problems.” The prospective teachers also described learning as taking place via receipt from external expert sources, and this expertise included the course instructors, tutors, textbooks, and class notes. A student noted: "I even had people trying to tutor me to try to help me and the people that were tutoring me were having trouble and saying 'I don't understand what they're trying to ask you'… I ended up getting a tutor for [course] because I didn't understand it all. It was just too much."

Learning was also typified in the courses as "passing the course." The students spoke of "passing the test" and "getting in and getting out." For example, a student stated:

> It was here's the content you need to learn, here's the information, here's the test . . . I felt like it was just a core class that had to be taken, that you have to get through to move on . . . We're thinking we're learning something about how to be teachers but in reality, we're learning how to get through their math course.

Another asserted, "I can't really say that I learned anything from those classes except learning what you need to know to pass the test and pass the class and get out, which is kind of upsetting to me."

Teaching mathematics. The teaching of mathematics in the typical courses was typified as explaining, and the prospective teachers experienced teaching as "lecturing," "showing," "step-by-step explanations," "Power Points," and "covering content." Further, teaching in the courses was characterized as teacher-centered and content-centered rather than attentive to the needs of the students. The teaching was frequently described as "fast-paced." A student asserted, "I wouldn't just use a Power Point, and then having students copying, and you can't print out the slides . . . But my problem was after three hours I was getting dizzy from looking at the overhead to try to copy so far. For me, I need to take notes so I can remember and do it but it just went so fast." Another described the classrooms as "rows of desk" and "Power Point slides were the instruction mostly. [I] must be quick with the hand to take these notes. It was a lecture based course, every one of them . . . It was very we've got to hurry up and move on . . . make sure we cover everything." Further, another student explained, "They [instructors] had to cram a lot in . . . we've got to get through this . . . questions were asked, very rarely answered completely."

Qualitative Findings: Alternate Courses

Knowing mathematics. The students portrayed mathematics in the alternate courses as process-focused, useful, challenging, and internally constructed. The process focus included an emphasis on "problem solving" and "understanding," which contributed to flexibility in their mathematical knowledge. For example, a student explained, "We talked about the process . . . how everything worked . . . it was delving deeper into how exactly do you solve this problem." The mathematics was also described as "useful" or "relevant," with explicit connections to the mathematics in the elementary classroom. A student described the usefulness of the courses as:

> I think you can really gear your lesson plans more around the needs of the child if you really know what they're thinking and their initial approach to it versus just being told here's the formula, here's how you do it . . . I think it just makes a different, a complete difference in math for kids.

In addition, another stated, “I actually saw how a kindergartener does statistics and probability . . . I’m actually using it and putting it into [teaching] practice.”

Further, the mathematics was typified as “challenging;” it was a “struggle” for the prospective teachers to “unlearn” mathematics as being simply procedures. A student stated, “It was challenging . . . I told a lot of my friends that I was in an elementary math class and they were like ‘Oh yeah, that must be really easy,’ but it was definitely challenging.” One provided this example:

> It was at first a bit challenging to actually get into the mind of a child and try to kinda figure out how they would think about it. I’ve never taken a math course like that before . . . And it was difficult at first to see that there were 11 different addition, subtraction problems. I thought it was addition, subtraction.

Another described the mathematics in the courses as:

> The difficult part is taking away the memorization and the recall the way that I learned mathematics . . . I know the procedure, I don’t need to understand how to explain that procedure. So, really having to understand and comprehend and having to do the higher levels of thinking and taking that discourse and turning it into understanding were the tough parts.

Mathematics in the courses was also portrayed as internally constructed rather than received from other external expert sources. For example, a student stated:

> Even though those [mathematics] classes ended, I still have everything I learned, and I don’t have any books in front of me. But, I have it in my head, and I’m able to bring it out when I need to. Then I can go into a [elementary] classroom . . . I can pull it together and teach that child, you know, the connection between multiplication and division without having to use a teacher guided book.

Further, in describing the mathematics in the courses, the students often contrasted it with the mathematics learned in other content courses. For example, one asserted, “My other math classes, I could just whip out a worksheet, show my work, and turn it in . . . It’s [mathematics classes] a lot more work in the sense that you have to show that you know math . . . You were working out problems you had never seen before and explaining why.” Further, another stated:

> I guess first walking into a math class you expect it to be, you know, paper pencil, a whole bunch of formulas, solving problems, and it was nothing like that actually. It was much more, it was much more useful. It was geared more toward how kids view math and how children learn math versus making me actually have to learn formulas and procedures.

Learning mathematics. The learning of mathematics in the courses was typified in several ways by the students. They described learning as occurring through a community of learners, with an emphasis on discourse. Further, learning took place through mathematical processes such as “problem solving,” which were portrayed as “engaging” and perceived as “okay to be wrong.” A student provided this illustrative statement:

> A good portion of the class we problem solved . . . We all usually problem

> solved and went through our problems in our groups, and we would share our responses with the class . . . It was actually much more of a community feeling . . . We were always talking and engaged and really working together . . . We weren't afraid to stand up and you know, share something when we didn't get it.

Learning mathematics also took place through a focus on children's thinking. The students explained, "We were always looking at how kids were doing this and really looking at their strategies;" and "How might a student solve it [math problem]? It made me more comfortable knowing that I would be able . . . to assist my students. So as a learner myself . . . I'm learning from my students." This study of children's thinking about mathematical ideas, including course assignments with a similar focus, led to learning as being perceived as directly applicable to the elementary classroom. A student explained, "It [class] all made sense, thinking like a child is supposed to think, and how I'm supposed to teach the mathematics." Another student stated:

> It [class] also showed me a hundred different ways that my kids are going to do it . . . It [class] was a kind of a two-fold thing then, because it rebuilt my content knowledge while showing me ways that kids will solve it and how to handle that. And, I guess what to expect when they [elementary students] get a division problem like that.

Teaching mathematics. Teaching in the context of the alternate courses was typified by the students as "guiding" and "questioning." One explained:

> I liked the way that it was taught to us the way that we would teach it in our [elementary] classrooms, and that everything was activities that the kids would have to do themselves. So, once I knew that I struggled and that I pulled through it [the mathematics], that they [elementary student] would eventually get to that point. When I asked a question [in class] I didn't always get the answer, I just got another question.

The instructor promoted discourse, created a safe learning environment, and used representations (e.g., manipulatives) relevant to the elementary classroom. One stated:

> Doing it [mathematics] was a lot more hands-on, and everyday [the instructor] would bring in cart-fulls of manipulatives and different tools and really show us how we can do this in our classroom and make a more meaningful experience for kids.

Interestingly, the students described a "struggle" or tension in connecting what they were learning in the courses to their teaching in their field placement classrooms, which were often characterized as "traditional." One student described her experience in this way:

> From what I could observe in my placement in my student teaching, there's a lot of great math being taught not in the best manner. I see a lot of . . . disconnected steps. There's no continuity in their lessons. And, then I'll look on the kids' faces, and all you see is all these question marks, and they're not good question marks. They're "I don't get where this is going, I don't see where we came from."

Similarly, another said:

> I loved my student teaching placement . . . But the math kind of was one thing where I just kind of gritted my teeth and stood back . . . The math class was: “Okay, today we’re doing long division. Here’s what you do, divide, multiply, subtract, bring down. Draw this cute little face. Do your steps.” When you’ve got that, when you’ve mastered that, um, algorithm I guess, then you were given a sheet of problem solving to work on . . . That’s sadly what I saw in almost all of my, you know, [elementary classroom] placements, was just this drill and practice. And it’s just, I need my own room so I cannot do that.

Discussion

Within a milieu of increasing recognition of the importance of mathematical knowledge of elementary teachers and the accompanying need for specialized mathematics content courses during teacher preparation, examining course experiences with the aim of determining efficacious ways of promoting student learning and change is paramount. The findings of this multiple case study, gleaned from a small sample typical of such a design, serve a descriptive purpose for the particular context and are not intended to support generalizations. However, the voices of the students in the two different mathematics content course experiences provide insights into the struggles considered inherent to this population as they learn mathematics and also pose considerations for course content, pedagogical approaches, and learning activities. The results also confirm and extend the extant literature.

In considering the first research question, there were differences in SCK and mathematical beliefs for these two groups of students upon completion of the teacher preparation program. The LMT mean scores revealed the students in the alternate courses had greater SCK than those in the typical courses and there was less variability in these scores. Two subscales, Number and Operations and Geometry, showed notable differences, thus suggesting more developed SCK in these content areas. The interview data extend these findings related to SCK. First of all, the students in the alternate courses typified the mathematics they were learning as difficult, thus acknowledge the arduousness of developing SCK and also challenging the perception of some that the mathematics in these courses is “easy”. They described a productive struggle to understand mathematics at deeper levels than their past learning of mathematics as simply computational procedures. Secondly, for the students in the typical courses, they believed they possessed the mathematical knowledge needed for teaching in the elementary classroom before taking the courses or even entering the university, similar to what others have asserted as a barrier for developing SCK (CBMS, 2012). The complexity of knowledge needed for teaching elementary mathematics, as embodied in SCK, was not realized as their course experiences did not challenge their paradigm about the necessary teacher competencies for effectively teaching elementary mathematics. It has been asserted that studying children’s thinking during mathematics content courses challenges students’ understandings about mathematics and leads to the recognition that their own mathematical understandings are insufficient for teaching elementary mathematics (Philipp et al., 2007).

Also, in regard to mathematical beliefs at the end of the program, there were at least half point Likert scale differences in mean scores between the two groups on all subscales of the MBI. For the students in the alternate courses, more so than those in the typical courses, they believed children can construct their own mathematical knowledge and teaching should be organized to facilitate this construction, as well as that mathematical skills should be taught in relation to problem solving and understanding. Further, on both subscales of the MTEBI there were at least half point Likert scale differences in mean scores, thus suggesting the students in the alternate courses had stronger beliefs in their skills and abilities to teach mathematics effectively and influence student learning. Bandura (1986) postulated that efficacy beliefs are formed during experiences with a task; successful performances strengthen these beliefs while failures lower them. In the comments of the students in the alternate courses, it was evident as they engaged in mathematical tasks there was a growing sense of confidence in themselves as mathematical reasoners. On the MTEBI, the subscale that had the largest mean score difference was related to mathematics teaching outcome expectancy. Perhaps the emphasis in the alternate courses on observing and analyzing children's thinking about mathematics ideas and innate mathematical capabilities via videos and interviews may have contributed to this sense of confidence in impacting students' learning.

When considering the second research question, the two groups of students characterized their experiences with *knowing*, *learning*, and *teaching* mathematics in different ways. The students' descriptions of *knowing* mathematics in the context of the course experiences ranged from mathematics as a *record of knowledge*, characterized as elusive and difficult, to mathematics as processes, with a focus on problem solving and understanding. Differences such as these might be linked to the philosophical dissonance amongst some instructors of these courses and even departments about what constitutes the knowledge needed for teaching mathematics in the elementary classroom.

Further, when considering the descriptions of mathematics and course experiences, the importance of usefulness resonated across the comments. Both groups of students longed to situate the course experiences within their development as elementary teachers and to have connections to the elementary classroom. Given that much of their teacher preparation coursework is applied in nature, this is not unreasonable. These perceptions of relevance directly influenced the students' levels of motivation and satisfaction about the mathematics courses. However, it is noteworthy the students in the alternate courses expressed a marked tension in always connecting what they were learning in the courses to their teaching in their field placement classrooms. Often they described the learning and teaching of mathematics in their field placement classrooms as traditional in nature and anticipated what they would do differently in their own classrooms.

The findings related to *learning* and *teaching* in the course experiences revealed the students in the alternate courses valued a classroom environment based on a community of learners. Such a climate included an emphasis on dialogic discourse through guiding and questioning, with perceptions of safety when taking mathematical risks. Other appreciated pedagogical approaches included an emphasis on mathematical processes, with frequent mention of problem solving, as well as representations and tools relevant to the elementary classroom, with frequent reference to manipulatives. This focus seemed to assuage negative affect (i.e., beliefs, attitudes, emotions) toward mathematics held by the students. It has been found that mathematics content courses with an emphasis on mathematics as a sense-making activity and a focus on problem solving, reasoning, and justification increase elementary prospective teachers' confidence in their mathematical abilities (CBMS, 2012; Emenaker, 1996; Liljedahl, 2005; Lubenski & Otto, 2004).

Not all elementary teachers love mathematics, but they do love children, so studying children's thinking is a natural avenue for drawing students into a subject toward which they are too often anxious (Philipp, 2007). Providing connections to something to which they are fundamentally concerned, children, prompts motivation, learning, and change (Philipp, et al 2007). This proved to be true in this study, as the findings revealed a focus on children's thinking was important to learning and teaching in the alternate courses, and such an emphasis directly contributed to perceptions of relevance and compelled the students into the mathematics. It allowed the students to explore the mathematics from a fresh perspective, exposing their own misconceptions and build a deeper understanding of fundamental arithmetic operation. The focus on children's thinking was also motivational and helped them appreciate how important it is for them to know the content for their future roles as teachers. The applicability of the mathematics they were learning to their chosen career path was evident.

Considerable challenges exist in the mathematical preparation of elementary teachers, including a tendency to dislike and avoid mathematics, a view that teaching mathematics involves clearly explaining procedures, and a propensity to think there is nothing else for them to learn about the content of elementary school mathematics (CBMS, 2012). Instructors of mathematics content courses need to recognize that the messages of their courses may be filtered through such views. A further complication related to these courses is their housing varies across institutions of higher education, sometimes in departments of education and sometimes in departments of mathematics, with instructors as elementary educators, mathematics educators, or mathematicians. Clearly there is a need for articulation between departments and instructors about what the mathematical content focus should be, along with research-based pedagogical approaches that support the learning needs of this population. The findings of this study offer insights into course features and learning activities that prompted motivation, learning, and change. Notably, students understanding the relevance of the mathematics is paramount.

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